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XV.

ATMOSPHERIC REFRACTION.

BY EDWARD C. PICKERING.

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PART I.

DURING the summer of 1876 several thousand observations of the altitudes of the White Mountains were made by the writer.* The method of zenith distances was employed, the instrument used being the micrometer level. Great difference of opinion exists among geodetists regarding the value of the zenith distance of a distant object as a means of determining its height, on account of the uncertainty in the atmospheric refraction. A large number of measures of this quantity were accordingly made in subsequent years, to show to what extent the observations of 1876 were likely to be affected by this error. Four micrometer levels have been employed, which will be designated as A, B, C, and D.

The first of these, A, was shown to this Academy on January 11, 1876,† and was exhibited at the International Exposition of 1876 in Philadelphia. It consisted of a telescope conveniently mounted, carrying a level, and having in the eyepiece a spider-line micrometer.

Instrument B, which was that used in 1876, was made out of an architect's level. It consisted of a telescope having an aperture of 3 cm. and a focal length of 25 cm., with an erecting eyepiece magnifying eighteen diameters. A delicate spirit-level was attached directly to the telescope, which rested in two wyres, 19 cm. apart. One end of the bar carrying these wyres rested upon a bearing, and the other upon the pointed end of a micrometer screw. The bearing and the nut of the screw were carried upon a bar which rotated around a vertical axis. A horizontal circle divided into degrees was added, for

* Appalachia, I. 138.

† Proc. Amer. Acad., XI. 256.

convenience in identifying the objects observed. The pitch of the screw was about 0.1 cm., and its head was divided into hundredths. The value of one division in seconds of arc was found by placing the instrument upon the cube of the telescope of the Meridian Circle of the Harvard College Observatory. A series of readings was then taken, setting the screw in various positions, and inclining the cube until the bubble showed that the small telescope was horizontal. The position of the bubble was then read, and the inclination determined by the large circle of the Meridian Circle. The value of one division was thus found, on June 7, 1877, to be $13''.95$. As the instrument is used only for small altitudes, the angles are practically measured by an accurate tangent screw on a circle about 38 cm. in diameter.

Instrument C is larger than B, and the cross-hairs in the latter are replaced by a filar-micrometer. It therefore combines the advantages of A and B. Its telescope has an aperture of 4 cm., a focal length of 45 cm., and a magnifying power of sixteen diameters.

Instrument D closely resembles B. Light is secured by the use of an inverting eyepiece. The aperture of the telescope is 2.5 cm., its focal length 21 cm., and its magnifying power ten. The value of one division of the screw was found as described above, and gave the result $13''.95$, or the same as B.

An important improvement was devised by Mr. J. R. Edmands, and introduced in this instrument, in March, 1882. The level was attached to the bar carrying the wyes, instead of to the telescope. As the instrument was originally used, the observations were rendered differential, the quantity measured being the difference in apparent altitude of the various objects. The only quantity liable to vary in using the instrument is the angle between the true level line and the axis of the instrument when the bubble of the level is the centre. This angle may be divided into two parts: first, the collimation error due to the deviation of the axis from the line passing through the centre of the pivots by which the telescope is supported in its wyes; and, secondly, the level error, by which this last line deviates from the horizontal when the bubble is in the centre. Each of these constants may be determined by the device proposed by Mr. Edmands. The level is attached to the bar carrying the wyes, instead of to the telescope. If now the telescope is turned around its axis 180° , the mean of the readings in the two positions will eliminate the collimation. The value of this constant is equal to one half the difference in these readings. The level error is similarly eliminated or determined by turning the telescope end for end around a vertical axis. Rotating the telescope around a horizontal

axis perpendicular to its own axis, gives both constants together, without distinguishing between them. The direction of the three axes around which the telescope should be rotated to give the collimation, level, or both combined, may be remembered as the axis of the telescope, the vertical cross-hair, and the horizontal cross-hair, respectively. With this modification, the micrometer level compares favorably with the vertical circle as regards economy in time and expense, portability, and accuracy when small angles are to be measured.

Before this modification, the difficulty in determining the combined level and collimation error was avoided by observing the difference in height of a distant and near mountain which are nearly in line. These were commonly so selected as to be in the same field of view. It was therefore only necessary to observe the difference in altitude by the micrometer screw under various atmospheric conditions.

The first series of measurements were made on Mt. Wachusett, in Princeton, Mass., on June 22, 1877. Accompanied by Col. C. W. Folsom, I reached the summit at about noon. The observations were begun at once with instrument B, and were continued with short intermissions until nearly eight o'clock, when darkness interfered. The following morning was exceptionally clear, and observations were begun at a quarter after four, and continued until two in the afternoon, when we left the summit.

Mt. Monadnock, Cheshire Co., N. H., was visited, July 3, 1877, by Mr. J. R. Edmands, Professor C. E. Fay, and myself. We reached the summit at about three in the afternoon, and began observing with instrument B. The air was so hazy that the Sandwich range was barely visible, and Ascutney, the Southern Kearsarge, and Gunstock were the most distant points observed. The observations were abandoned at about five o'clock on account of the increasing haze. On July 15, 1877, Mr. Edmands and I visited Mt. Washington, in the White Mountains, where we remained until July 24. The almost continuous cloud and haze prevented much work, except on July 18, when a few preliminary observations were made with instrument C; and on July 21 and 22, when observations were obtained with D. Mt. Kearsarge, Merrimack Co., N. H., was occupied at the same time by Mr. J. B. Henck, Jr., with instrument A, in order that simultaneous observations might be obtained at the ends of the line from Mt. Washington to Kearsarge.

Observations were undertaken in August and September, 1877, at Jefferson Hill, N. H., by Mr. Henck, and at East Jefferson, by Mr. Edmands, with instruments B and D, respectively. Again long

periods of storm and haze prevented the accomplishment of satisfactory work.

Some observations were obtained by Mr. Edmands from Mt. Starr King, Coos Co., N. H., in October, 1878. In November and December of the same year, Mr. Edmands resided at Arlington Heights, Mass., for the purpose of obtaining similar observations.

Plans were also made for occupying simultaneously for a week Mts. Moosilauk, Kearsarge (S.), and Wachusett, which lie nearly in a straight line. Observations could thus be obtained at the ends and near the middle of a line nearly a hundred miles in length.

Much time was lost, owing to bad weather, when stations were occupied which were especially adapted to the determination of the atmospheric refraction. That a very distant point might be seen, lofty and comparatively inaccessible stations were required. Moreover, in order that observations might be made under the extremes of temperature and other atmospheric conditions, it was necessary that the work should extend over a considerable period of time. A series of measurements was accordingly undertaken at the Harvard College Observatory, where the absence of very distant points was compensated by the ease with which observations could be made at all times and seasons without materially interfering with other work. The observations were begun on January 3, 1882, and continued until January 26. During this series the temperature fell as low as -15° F. The position of the level was then changed, as described on page 269. A second series extended from May 6 to August 7, 1882, and included observations in which the temperature was $+91^{\circ}$ F. All the observations were made with instrument D, by Mr. Edmands and myself.

Only the observations made at Wachusett and at the Observatory will be considered in the present paper.

I. WACHUSETT.

The micrometer level was mounted on the sill of the north window of room No. 19, in the second story of the hotel, on the summit of Wachusett. This point is approximately 15m. north, 3m. west, and 4m. above the United States Coast Survey bolt in the top of the mountain. The effect of the unsteadiness in the support is almost entirely eliminated by the differential character of the observation. The observations were made by directing the telescope towards the mountain to be observed, and turning the screw so that the level-tube was nearly horizontal. The position of the screw, of the two ends of

the bubble, and the time, were then recorded. The telescope was next directed alternately to the distant and near mountains, five times to each, and the screw readings recorded. The level was then again observed. These measures constituted one set. Sometimes a third mountain was also observed, and sometimes the level was omitted. Care was taken never to move the telescope horizontally during a set, and when several points were observed, they were always measured in the same order.

Table I. gives in the first column the Greenwich mean time, expressed in hours and tenths, the beginning of the day being taken at Greenwich noon. The second column gives a designation for the object pointed at; and the third gives the corresponding mean of the readings of the micrometer screw. L is used to denote that the telescope was level, and the reading in the third column is here corrected for the position of the bubble. The objects belonging to the same set are indicated by placing the time opposite the first only.

The various designations in the second column have the following meanings:—

B. Barrett Hill. Top of trees on hill nearly in line with Moosilauk and Kearsarge (S.). E. Mt. Equinox. G. Gunstock. Gp. Gap Mt. Gr. Greylock. J. Joe English Hill. K. Kearsarge (S.). The object observed was the base of the house, a short distance from the summit. This is apparently the house which has been destroyed, and not that now standing a few feet only from the summit. Ki. Killington Peak. L. Level, as stated above. M. Moosilauk. Mn. Monadnock. P. Passaconaway. Pi. Piscataquog, the hill nearly in line with Whiteface. W. Whiteface. 1. Summit of Kearsarge (S.). 2. Top of house on Kearsarge (S.). 3. Summit of a cloud-dome, presumably over Mt. Washington. 4. Northwestern of the Uncanoonucs. 5. Southeastern of the Uncanoonucs. 6. Cupola of Barnard House, Arlington Heights. 7. Roof of barn placed in right-hand part of field to determine inclination of wire. 8. Roof of barn in left-hand part of field. The two sets in which this object was observed showed that the inclination of the wire equals 0.6 division for $1^{\circ}.1$, or about $8''$ per degree, including error in level.

TABLE I.—JOURNAL.

G. M. T.	Object.	Setting.	G. M. T.	Object.	Setting.
h.			h.		
4.8	L.	142.36	12.1	G.	249.78
"	Mn.	72.40	"	J.	254.40
4.9	L.	140.77	12.2	G.	255.08
"	L.	27.80	"	J.	259.94
5.0	L.	138.97	12.3	W.	50.38
"	K.	227.91	"	S.	68.2
5.9	L.	150.04	"	Pi.	66.95
"	Mn.	80.72	12.3	G.	251.46
6.0	L.	148.31	"	J.	256.04
"	K.	237.22	12.5	G.	52.78
6.1	L.	146.97	"	J.	57.54
"	G.	259.16	12.5	W.	57.22
"	J.	261.74	"	Pi.	72.06
6.2	L.	150.45	"	G.	51.42
"	Ki.	230.78	"	J.	55.94
"	Gp.	256.16	21.0	L.	158.44
6.3	Ki.	228.96	"	G.	62.42
"	Gp.	255.01	"	J.	69.30
6.4	G.	256.02	21.1	L.	159.03
"	J.	258.74	"	P.	63.76
7.1	G.	255.70	"	W.	62.28
"	J.	258.88	"	Pi.	81.42
7.2	G.	256.14	21.2	G.	61.84
"	J.	259.18	"	J.	68.58
7.2	K.	231.06	21.6	G.	60.94
"	B.	244.44	"	J.	67.58
7.3	K.	231.18	21.7	L.	158.03
"	B.	243.82	"	P.	262.54
10.1	L.	154.98	"	W.	260.72
"	Mn.	85.00	"	Pi.	279.88
10.2	L.	150.97	21.8	L.	157.92
"	K.	264.74	"	M.	256.14
"	B.	276.48	"	K.	265.50
10.3	1.	261.46	"	B.	279.08
"	2.	262.70	21.8	L.	157.71
"	K.	263.72	"	Mn.	87.90
"	B.	272.70	21.9	L.	159.24
10.4	G.	256.04	"	Mn.	88.34
"	J.	260.10	22.0	L.	157.91
10.4	K.	233.98	"	Ki.	225.34
"	B.	247.70	"	Gp.	256.06
10.8	G.	261.08	22.2	L.	158.97
"	J.	265.22	"	M.	257.57
10.8	G.	260.02	"	K.	264.94
"	J.	263.74	"	B.	278.02
11.6	G.	253.82	22.3	L.	158.30
"	J.	258.26	"	P.	262.36
11.7	K.	257.02	"	W.	260.52
"	B.	268.34	"	Pi.	279.88
11.8	W.	255.76	22.3	L.	157.34
"	Pi.	269.92	"	G.	260.96
11.9	G.	255.18	"	J.	267.84
"	J.	259.34	23.0	L.	151.68
12.1	G.	249.08	"	Gr.	192.38
"	J.	253.80	23.1	L.	154.36

TABLE I.—*Continued.*

G. M. T.	Object.	Setting.	G. M. T.	Object.	Setting.
h.			h.		
23.1	E.	175.70	4.2	Pi.	281.28
23.2	L.	157.55	4.3	L.	154.19
"	P.	261.88	"	G.	264.52
"	W.	259.76	"	J.	268.42
"	Pi.	278.74	4.4	L.	154.12
23.3	L.	158.24	"	G.	265.36
"	M.	259.57	"	J.	268.06
"	K.	265.13	4.5	L.	155.36
"	B.	278.46	"	M.	286.40
23.4	L.	156.76	4.6	L.	153.44
"	G.	261.52	"	J.	267.20
"	J.	267.20	"	4.	264.75
23.6	L.	157.27	"	5.	265.90
"	G.	262.54	4.8	L.	152.96
"	J.	268.08	"	G.	264.32
0.7	L.	156.97	"	J.	267.48
"	G.	265.22	4.9	L.	152.16
"	J.	269.22	"	G.	263.40
0.9	L.	157.87	"	J.	266.84
"	Mn.	86.96	5.4	L.	179.24
1.0	L.	157.92	"	6.	366.44
"	Mn.	87.16	5.6	L.	144.92
1.0	L.	156.92	"	G.	255.54
"	P.	269.12	"	J.	259.76
"	W.	266.08	6.2	L.	148.97
"	Pi.	281.24	"	G.	260.52
1.1	L.	156.29	"	J.	263.64
"	G.	265.14	6.2	L.	151.08
"	J.	268.74	"	Mn.	81.20
1.2	L.	155.81	6.3	L.	148.97
"	G.	265.74	"	K.	263.28
"	J.	269.62	"	B.	273.94
1.4	L.	155.45	6.4	L.	148.87
"	J.	269.04	"	P.	264.78
"	4.	266.49	"	W.	262.02
"	5.	267.23	"	Pi.	274.90
2.6	L.	153.88	6.4	L.	149.60
"	G.	264.22	"	Ki.	228.44
"	J.	267.98	"	Gp.	254.22
2.8	L.	153.35	6.5	L.	148.87
"	G.	263.81	"	G.	259.60
"	J.	266.78	"	J.	263.26
4.2	L.	155.34	6.6	7.	29.84
"	G.	265.44	"	8.	29.24
"	J.	268.80	"	L.	149.56
4.2	L.	155.18	"	G.	261.38
"	P.	269.26	"	J.	264.12
"	W.	267.64			

Besides the above settings, the altitudes and azimuths of about thirty mountains were taken on June 22 between 6^h.4 and 7^h.1, and of about forty more on June 23, between 1^h.4 and 2^h.6. The total number of pointings of the telescope was 816, and of level readings

262, although the entire stay upon the mountain little exceeded twenty-four hours.

The most complete series of measurements relates to the comparative altitudes of Gunstock and Joe English Hill. Table II. gives in successive columns, first, the Greenwich time in hours and tenths; secondly, the height of the barometer reduced to 32° F. by means of the reading of the attached thermometer; and, thirdly, the temperature of the outer air in Fahrenheit degrees. These quantities are taken from curves drawn to represent the observations which were made at intervals during the day. The fourth column gives the zenith distance of the sun in degrees uncorrected for refraction. The next column gives the apparent height of Gunstock above Joe English in seconds of arc, being the observed micrometric distance multiplied by $13''.95$, the value of one division in seconds. From the assumed heights of the two hills and the apparent difference in altitude, the coefficient of refraction has been computed by Mr. Edmands, and is given in the last column but one of the table. A smooth curve was constructed with these values, and the last column gives the residual found by subtracting the result given by the curve from that given by observation. Any error in the assumed heights of the three points would alter the coefficient of refraction by an amount constant throughout the table, but would not affect the amount of its variation.

Several important conclusions may be drawn from Table II. The angular interval between the two hills gradually increased, and with it the coefficient of refraction gradually diminished, during the first afternoon. The changes were small during the last half-hour, notwithstanding the cooling of the air in the upper regions of the atmosphere, and other important meteorological changes caused by sunset. The following morning the refraction was much less, increasing during the morning, and becoming somewhat irregular during the day. These changes are much greater than the errors of observation, as is shown by the accordance of successive sets, especially on the afternoon of June 22.

Table III. gives the comparison of the observations of Passaconaway, Whiteface, and Piscataquog. The first column gives the time, and the second the angular interval between the first and third of these mountains expressed in seconds of arc. The next column gives the corresponding coefficient of refraction, and the fourth gives the residual found by subtracting from this coefficient the corresponding ordinate of the smooth curve deduced from the observations given in Table II. The last three columns give the corresponding quantities for the line Whiteface and Piscataquog.

TABLE II. — GUNSTOCK AND JOE ENGLISH.

G. M. T.	Barom.	Therm. F.	Z. D.	G. — J.	Coef. Refrac.	O. — C.
h.	in.	°	°	"		
6.1	27.62	62	25	36	.0708	+ .0001
6.4	27.62	62	28	38	.0707	+ .0001
7.1	27.64	61	35	44	.0702	— .0001
7.2	27.64	61	36	42	.0704	+ .0001
10.4	27.69	57	70	57	.0693	— .0003
10.8	27.71	56	75	58	.0693	— .0002
10.8	27.71	56	75	52	.0697	+ .0003
11.6	27.73	54	83	62	.0690	— .0001
11.9	27.74	54	87	58	.0692	+ .0002
12.1	27.74	54	89	66	.0688	— .0001
12.1	27.74	54	89	64	.0688	— .0001
12.2	27.74	54	90	68	.0685	— .0003
12.3	27.74	54	91	64	.0688	.0000
12.5	27.75	53	93	66	.0686	— .0001
12.5	27.75	53	93	63	.0689	+ .0002
21.0	27.87	46	94	96	.0665	— .0001
21.2	27.87	47	92	94	.0667	+ .0001
21.6	27.88	48	88	93	.0667	+ .0001
22.3	27.90	50	80	96	.0665	— .0003
23.4	27.91	53	68	79	.0677	.0000
23.6	27.92	53	67	63	.0678	.0000
0.7	27.94	55	55	56	.0694	+ .0001
1.1	27.94	56	50	50	.0698	+ .0003
1.2	27.94	56	48	54	.0695	— .0001
2.6	27.95	57	34	52	.0696	— .0003
2.7	27.95	57	32	41	.0704	+ .0004
4.2	27.93	59	19	47	.0700	— .0001
4.3	27.93	59	19	54	.0695	— .0006
4.4	27.93	59	19	38	.0707	+ .0006
4.8	27.92	59	19	44	.0702	+ .0001
4.9	27.92	59	19	48	.0700	— .0001
5.6	27.90	59	21	59	.0692	— .0009
6.2	27.89	60	26	44	.0703	+ .0002
6.5	27.88	60	29	51	.0697	— .0004
6.6	27.88	60	30	38	.0707	+ .0006

TABLE III. — PASSACONAWAY, WHITEFACE, AND PISCATAQUOG.

G. M. T.	P. — Pi.	Ref.	O. — C.	W. — Pi.	Ref.	O. — C.
h.	"			"		
11.6	198			224	.0670	— .0021
12.3				231	.0667	— .0021
12.5				207	.0677	— .0010
21.1	243	.0669	+ .0003	267	.0652	— .0014
21.6	242	.0671	+ .0005	267	.0652	— .0014
22.5	244	.0670	+ .0002	270	.0651	— .0017
23.2	235	.0674	.0000	279	.0647	— .0027
1.0	169	.0700	+ .0005	211	.0675	— .0020
4.2	168	.0701	.0000	190	.0684	— .0017
6.4	141	.0712	+ .0010	180	.0688	— .0013

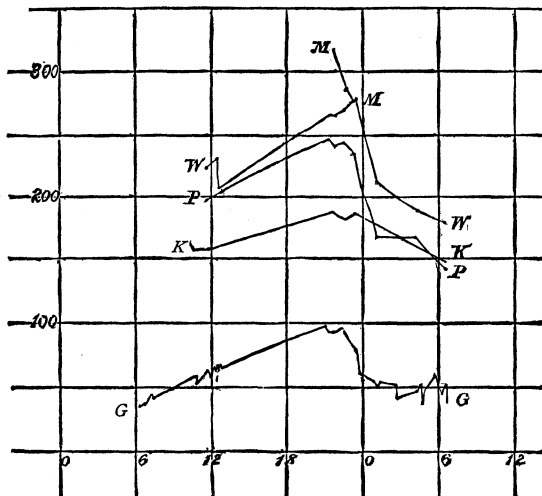
TABLE IV. — MOOSILAUKE, KEARSARGE, AND BARRETT HILL.

G. M. T.	M. — B.	Ref.	O. — C.	K. — B.
h.	"			"
10.2				164
10.3				157
11.7				158
21.6	319	.0697	+ .0030	189
22.5	288	.0710	+ .0043	182
23.3	278	.0713	+ .0037	186
6.3				149

Table IV. gives the corresponding results for Moosilauk, Kearsarge, and Barrett Hill. The last two columns are omitted, since the height of Barrett Hill is not known with sufficient precision to compute them.

The height of the station occupied on Wachusett is 620 meters. The heights and distances of the various points considered in Tables II. and III. are as follows :— Gunstock, height, 730 m., distance, 121.5 km. Joe English Hill, height, 401 m., distance, 54.1 km. Passaconaway, height, 1280 m., distance, 167.5 km. Whiteface, height, 1221 m., distance, 164.7. Piscataquog, height, 368 m., distance, 48 km. Moosilauk, height, 1466 m., distance, 170.3 km. Kearsarge (S.), height, 897 m., distance, 99 km.

The general agreement of these results is shown in the following diagram, where abscissas represent times, and ordinates the angular distance given in Tables II., III., and IV. The letters G, K, P, W,



and M respectively indicate Gunstock, Kearsarge (S.), Passaconna-way, Whiteface, and Moosilauk, as in Table I.

II. HARVARD COLLEGE OBSERVATORY.

The station occupied was on the southern corner of the western balcony of the Sears Tower of the Harvard College Observatory. Its height is about 30 meters above mean tide level. Three points were selected very nearly in line which are designated below as *A*, *B*, and *C*. *A* is the summit of Fenno's Peak, the second highest of the Blue Hills of Milton, and nearly the most distant terrestrial object visible from the Observatory. Its height is about 160 m., and its distance assumed to be 18.865 km. *B* is the top of the chimney of the house No. 20 Terrace Avenue, Roxbury District, Boston. Its distance is assumed to be 7.042 km. *C* is the top of the roof at the base of the lightning-rod of the gasholder on Everett Place, Cambridge. Its assumed distance is 0.700 km. *B* is about 6' below *A*, and *C* about 17' below *B*. Each set consisted of nine settings in the order *A B C A B C A B C*, the readings being made to tenths of one division of the micrometer screw. The mean of the three readings of *A* was then subtracted from the corresponding means of *B* and *C*. The various sets are enumerated in Table V. The successive columns give a number for reference, the date, and the hour and tenth in Greenwich mean time. The day is here assumed to begin at Greenwich noon, or at about seven o'clock in the morning of Cambridge mean time. The only occasion on which the day changes during a series of observations is between Nos. 120 and 121. The barometric pressure in inches after correction for the temperature of the mercury, and the temperature in degrees Fahrenheit, are given in the next columns. These are followed by the intervals *AC* and *BC* expressed in seconds of arc. One division of the micrometer is assumed to be equal to $14''.2$, in accordance with a determination made at that time. As the observations are differential, the uncertainty in the value of this constant will be inappreciable in the result. The following column gives the zenith distance of the sun in degrees. The next two columns give the residuals in seconds, found by subtracting from the observed value of *AC* its mean value, $1392''$, and also the same residuals after applying the correction for temperature and pressure to be described below. The last column but one gives the probable error of a single setting, expressed in seconds of arc, derived from the accordance of the individual readings. The last column gives the initial of the observer. E. indicates the observations of Mr. Edmands, and P. my own observations.

TABLE V.—JOURNAL.

No.	G. M. T.	Bar.	Th.	A C.	B C.	Sun.	Res.	Res.	P. E.	Obs.
	1882.	h.	in.	o	"	"	o	"	"	"
1	Jan. 3	3.5	30.01	+12	1406	1016	68	+14	— 5	3.3 P.
2	"	3.5	30.01	+12	1404	1008	68	+12	— 6	0.9 E.
3	"	8.3	29.98	+22	1404	1013	82	+12	— 9	3.2 P.
4	"	9.2	29.98	+20	1407	1014	88	+15	— 6	1.1 "
5	"	9.3	29.98	+20	1412	1016	89	+20	— 1	2.0 "
6	"	9.4	29.98	+19	1412	1013	90	+20	0	1.7 "
7	"	9.5	29.98	+19	1415	1010	91	+23	+ 3	1.7 "
8	"	9.6	29.98	+19	1420	1015	92	+28	+ 8	1.6 "
9	"	9.7	29.98	+18	1422	1016	93	+30	+10	2.3 "
10	"	9.8	29.98	+18	1423	1008	94	+31	+11	2.3 "
11	"	9.9	29.98	+17	1425	1020	95	+33	+14	5.3 "
12	"	10.0	29.98	+17	1450		96	+58	+39	7.5 "
13	Jan. 4	1.0	30.27	+ 3	1406	1002	77	+14	+ 1	3.3 "
14	"	2.0	30.30	+ 3	1405	1002	76	+13	0	1.8 E.
15	"	5.3	30.37	+ 6	1399	1006	67	+ 7	— 6	4.9 P.
16	"	6.1	30.39	+ 8	1404	1006	69	+12	— 2	3.0 "
17	Jan. 5	1.1	30.64	0		1040	82			4.0 "
18	"	5.2	30.68	+ 6	1423	1015	67	+31	+20	2.6 "
19	"	8.2	30.64	+20		1003	78			2.2 "
20	Jan. 7	0.9	29.85	+33	1405	1035	77	+13	—13	... "
21	"	0.9	29.85	+33	1410	1007	83	+18	— 8	2.7 "
22	"	1.5	29.87	+34	1407	1008	78	+15	—11	2.1 "
23	"	2.7	29.89	+36	1401	1005	70	+ 9	+16	2.4 "
24	Jan. 22	3.2	29.45	+38	1403	1010	65	+11	+15	2.4 "
25	"	4.4	29.46	+38	1391	1000	62	— 1	+ 3	0.9 "
26	Jan. 23	3.6	30.01	+ 4	1398	1004	64	+ 6	— 9	2.2 E.
27	"	5.6	30.01	+ 5	1389	1000	64	— 3	+16	2.1 "
28	"	7.5	30.07	+ 4	1392	999	75	0	—14	1.8 "
29	"	9.2	30.11	+ 3	1394	1000	84	+ 2	—11	1.8 P.
30	"	9.6	30.13	+ 3	1398	1002	88	+ 6	— 7	5.2 E.
31	"	9.7	30.13	+ 3	1394	1003	89	+ 2	—11	2.0 P.
32	"	9.7	30.14	+ 2	1397	1008	89	+ 5	— 8	2.9 "
33	"	9.9	30.14	+ 2	1405	1006	91	+13	0	1.6 E.
34	"	10.0	30.15	+ 2	1395	1000	92	+ 3	—10	2.2 P.
35	"	10.1	30.15	+ 1	1414	1013	93	+22	— 3	2.0 E.
36	"	10.2	30.16	+ 1	1387	996	94	— 5	—18	4.9 P.
37	Jan. 24	0.4	30.59	—15	1407	1005	84	+15	+11	4.6 "
38	"	0.6	30.59	—14	1414	1010	82	+22	+17	0.9 "
39	"	1.0	30.59	—13	1410	1004	78	+18	+13	2.6 "
40	"	1.1	30.60	—13	1412	1013	77	+20	+15	2.7 E.
41	"	3.2	30.66	— 8	1409	1008	64	+17	+11	2.7 P.
42	"	6.3	30.64	+ 2	1389	999	66	— 3	—13	3.7 "
43	"	7.2	30.66	+ 3	1398	1004	72	+ 6	— 5	2.6 "
44	"	9.2	30.67	+ 4	1402	1009	84	+10	— 1	1.4 "
45	"	10.0	30.70	+ 3	1413	1018	92	+21	+10	3.3 "
46	Jan. 25	0.3	30.62	0	1430	1012	84	+38	+29	1.9 E.
47	"	0.6	30.62	+ 1	1440	1010	82	+48	+38	5.6 "
48	"	1.7	30.61	+ 4	1424	1001	73	+32	+21	1.9 P.
49	"	8.9	30.42	+26	1405	1000	81	+13	— 7	2.2 E.
50	"	9.9	30.41	+25	1405	997	91	+13	— 7	2.1 "
51	"	10.1	30.41	+25	1415	991	93	+23	+ 3	2.1 "
52	Jan. 26	7.5	29.79	+42	1411	1015	74	+19	—10	1.8 P.
53	May 4	1.7	29.71	+59	1386	1001	48	— 6	+ 6	2.4 "
54	"	2.9	29.70	+62	1382	994	36	—10	+ 1	0.9 "

TABLE V.—*Continued.*

No.	G. M. T.		Bar.	Th.	A C.	B C.	Sun.	Res.	Res.	P. E.	Obs.	
	1882.	h.	in.	°	"	"	°	"	"	"		
55	May	4	3.8	29.68	+65	1384	1002	29	— 8	+ 2	4.4	P.
56	"		5.0	29.65	+67	1383	996	27	— 9	0	9.3	"
57	"		5.2	29.65	+67	1397	1012	27	+ 5	+14	2.7	E.
58	"		5.8	29.62	+69	1378	991	29	—14	— 6	3.0	P.
59	"		8.0	29.57	+68	1369	982	49	—23	—14	7.4	"
60	"		8.6	29.58	+66	1383	996	54	— 9	0	2.2	"
61	May	5	0.8	29.78	+45	1386	995	57	— 6	+11	2.4	"
62	"		3.6	29.81	+47	1388	1005	31	— 4	+13	6.4	"
63	"		5.5	29.79	+48	1376	1000	28	—16	+ 1	5.8	"
64	"		8.7	29.77	+49	1370	988	56	—22	— 6	5.1	"
65	May	6	2.6	30.04	+47	1381	984	39	—11	+ 7	1.0	"
66	May	31	1.6	29.79	+66	1373	996	45	—19	— 9	5.8	"
67	"		1.7	29.79	+66	1381	991	44	—11	— 1	7.0	"
68	"		4.1	29.75	+74	1391	1008	22	— 1	+ 7	3.6	"
69	June	2	2.0	29.76	+58	1372	995	41	—20	— 7	2.2	"
70	"		3.4	29.78	+62	1384	1009	27	— 8	+ 5	2.2	"
71	"		4.7	29.79	+65	1379	1008	20	—13	— 2	1.0	"
72	"		10.6	29.77	+69	1373	995	72	—19	—10	1.8	"
73	"		12.4	29.79	+64	1418	1025	92	+26	+11	3.1	"
74	June	5	8.0	29.55	+68	1370	1000	44	—22	—14	3.0	"
75	"		9.0	29.55	+66	1382	994	55	—10	— 1	1.4	"
76	"		12.1	29.59	+63	1390	997	89	— 2	+10	3.0	"
77	"		12.4	29.60	+62	1409	1015	92	+17	+18	4.8	"
78	June	24	4.8	29.88	+86	1377	991	20	—15	—11	2.6	"
79	"		5.4	29.86	+88	1367	973	21	—25	+13	2.4	"
80	"		6.4	29.83	+90	1364	977	29	—28	+ 9	3.7	"
81	"		7.4	29.83	+91	1368	982	39	—24	+13	2.4	"
82	"		8.4	29.81	+91	1361	975	50	—31	+ 5	2.1	"
83	"		9.4	29.81	+91	1372	986	61	—20	+16	3.6	"
84	"		10.3	29.81	+89	1380	986	70	—12	— 9	3.3	"
85	July	11	7.2	29.62	+91	1369	987	37	—23	+12	2.1	"
86	"		9.1	29.61	+89	1385	991	58	— 7	— 5	3.1	"
87	"		9.4	29.60	+87	1384	993	61	— 8	— 6	1.6	"
88	Aug.	6	2.5	29.97	+80	1387	994	38	— 5	+ 1	1.9	E.
89	"		3.9	29.94	+86	1382	995	27	—10	— 5	1.8	"
90	"		5.7	29.93	+91	1386	994	29	— 6	— 3	2.8	P.
91	"		5.8	29.91	+91	1372	989	29	—20	0	3.1	E.
92	"		6.7	29.91	+91	1381	982	36	—11	— 8	3.4	P.
93	"		7.9	29.90	+88	1382	989	48	—10	— 6	1.4	"
94	"		8.7	29.89	+87	1384	983	57	— 8	— 4	2.2	"
95	"		9.7	29.88	+88	1378	988	71	—14	—10	3.1	"
96	"		10.7	29.88	+86	1389	999	79	— 3	+ 1	1.9	"
97	"		11.4	29.88	+84	1398	1005	85	+ 6	+11	3.1	E.
98	"		11.6	29.88	+84	1401	1012	87	+ 9	+14	1.3	"
99	"		11.6	29.88	+83	1396	1009	87	+ 4	+ 9	2.1	"
100	"		11.7	29.88	+83	1397	1005	88	+ 5	+10	3.0	"
101	"		11.9	29.88	+83	1402	1007	90	+10	+15	3.0	P.
102	"		12.0	29.88	+82	1395	998	91	+ 3	+ 8	3.2	"
103	"		12.2	29.88	+82	1395	995	93	+ 3	+ 8	1.4	"
104	"		12.3	29.88	+81	1390	990	94	— 2	+ 4	1.1	"
105	"		12.3	29.88	+81	1384	997	94	— 8	— 2	1.4	"
106	"		12.4	29.88	+80	1364		95	—28	+12	6.8	"
107	"		21.4	29.85	+71		997	93			0.3	E.
108	"		21.6	29.85	+71		995	91			1.5	"

TABLE V. — *Continued.*

No.	G. M. T.	Bar.	Th.	A C.	B C.	Sun.	Res.	Res.	P. E.	Obs.
	1882. h.	in.	°	"	"	°	"	"	"	
109	Aug. 6	21.7	29.85	+71		993	90			2.3 E.
110	"	21.8	29.85	+71		994	89			1.7 "
111	"	22.2	29.85	+71		1001	85			2.3 "
112	"	22.3	29.85	+71		1000	84			1.7 "
113	"	22.4	29.84	+71		998	83			2.8 "
114	"	22.4	29.84	+71		996	83			2.2 "
115	"	23.1	29.84	+71	1394	997	75	+ 2	+11	2.8 "
116	"	23.2	29.78	+71	1396	1002	74	+ 4	+13	2.3 "
117	"	23.4	29.79	+72	1395	999	72	+ 3	+12	4.3 "
118	"	23.6	29.80	+72	1384	993	70	- 8	+ 1	3.0 "
119	"	23.7	29.80	+73	1388	993	68	- 9	- 1	1.3 "
120	"	23.8	29.81	+73	1387	997	67	- 5	+ 3	2.1 "
121	Aug. 7	0.0	29.81	+74	1389	997	65	- 3	+ 5	2.1 "
122	"	0.1	29.82	+74	1404	994	64	+12	+20	1.6 "
123	"	0.7	29.82	+76	1384	987	57	- 8	- 1	4.6 P.
124	"	1.7	29.80	+79	1387	987	47	- 5	+ 1	3.1 "
125	"	2.7	29.80	+83	1384	988	37	- 8	- 3	3.9 "
126	"	3.7	29.80	+85	1382	985	29	-10	- 6	1.2 "
127	"	4.7	29.78	+88	1385	987	26	- 7	- 4	1.7 "
128	"	5.7	29.76	+88	1379	981	29	-13	-10	1.9 "
129	"	6.7	29.73	+90	1362	975	37	-30	+ 6	2.0 "
130	"	6.8	29.72	+90	1368	974	38	-24	+12	1.6 "
131	"	7.4	29.72	+90	1365	974	44	-27	+ 9	3.2 "
132	"	7.7	29.71	+89	1367	985	47	-25	+12	2.3 "
133	"	9.5	29.70	+85	1392	1006	66	0	+ 4	0.7 E.
134	"	9.6	29.69	+84	1384	1002	67	- 8	- 4	2.2 "
135	"	9.7	29.69	+84	1384	1003	68	- 8	- 4	1.4 "
136	"	9.9	29.69	+83	1385	1005	70	- 7	- 3	1.8 "
137	"	10.1	29.68	+83	1385	1005	73	- 7	- 3	2.3 "
138	"	10.3	29.68	+82	1381	1002	75	-11	- 6	2.1 "
139	"	10.4	29.68	+82	1392	1009	76	0	+ 5	1.6 "
140	"	10.5	29.67	+81	1391	1003	77	- 1	+ 4	1.9 "
141	"	11.6	29.67	+81	1395	1001	87	+ 3	+ 8	1.6 P.
142	"	11.7	29.68	+80	1390	994	88	- 2	+ 3	2.6 "
143	"	11.8	29.70	+80	1388	994	89	- 4	+ 1	1.7 "
144	"	11.9	29.71	+79	1397	995	90	+ 5	+11	1.6 "
145	"	12.0	29.71	+79	1393	991	91	+ 1	+ 7	3.0 "

Additional evidence regarding the conditions under which these observations were made is furnished by the following notes.

1. Sun shining, snow on ground. Images somewhat unsteady.
12. *B* invisible, *A* and *C* difficult, owing to darkness.
15. Wind very high.
16. Image of *A* unsteady.
17. Hazy to south, but sun shining brightly.
20. The readings for *A* are 74.8, 79.7, and 98.8; for *B*, 03.6, 07.8, and 26.3; for *C*, 76.9, 81.2, and 98.2. The cause of this curious change is not certain. It was obvious to the eye, the cross-wires

appearing to withdraw from *A* after they had been set upon it. Ice had filled the holes in which the points of the tripod rested, and probably caused this motion. It may, however, have been due to a real change in the refraction, or to a change in the instrument, which had just been removed from a warmer place. In the subsequent sets the positions of *A*, *B*, and *C* remained constant near 91, 19, and 90.

24. Air unusually clear and steady. Wind high. Sun shining.
25. Cloudy.
26. Sun shining, wind high.
36. *C* difficult on account of darkness; settings discordant.
37. Air clear, wind light.
46. Hazy. *A* difficult.
53. Cloudy and a little hazy, but images good.
65. Images very bad.
74. Broken clouds, sun shining on *C* and Observatory.
75. Broken clouds, sun shining on *A*.
86. Heavy thunder-storm to the southwest.
87. *A* seen with difficulty owing to intervening rain.
107. *A* hidden in mist.

Fifty-two sets were taken during the winter, and gave the mean value of $AC = 1408''$, and $BC = 1008''$. The mean height of the barometer was 30.21 in., and of the thermometer 11° . Ninety-two sets were taken in warm weather, and gave $AC = 1384''$, and $BC = 995''$. Barometer 29.78 in., thermometer 76° .

The mean value of AC for the whole series was $1392''$, and that of $BC = 1000''$. The extreme range of the barometer was from 30.70 to 29.45 in., or 1.25 in. The thermometer varied from $+91^\circ$ to -16° , or 107° . The various values of AC differ from their mean of $1392''$ on the average by $16''.8$. But nearly all of the residuals of the winter observations are positive, those in summer negative. If the residuals of each series are taken from the mean of that series, the average value is reduced to $8''.5$. This is nearly the same as $8''.3$, the mean of the residuals after correcting for temperature and pressure. The probable error of a single setting is $2''.7$, both in the winter and summer observations. Since three pointings are made on each object, the errors of observation are much less than the deviations due to refraction. Moreover, some observations are included in which one or more of the objects were seen with great difficulty, on account of haze, unsteadiness of the air, or increasing twilight. Probably the error due to phase caused by varying illu-

mination, personal equation, and other causes, greatly exceeds this, especially on an object like a hill-top, on which there is no signal to be observed.

To study the nature of the variations they have been grouped in Table VI., according to the zenith distance of the sun, the height of the barometer, the height of thermometer, and the logarithm of the correction for temperature and pressure according to Bessel's formula for refraction. In each portion of the table the first column gives the approximate mean value of the argument in each group; the second, the number of sets; the third, the mean value of AC .

TABLE VI. — GROUPING OF OBSERVATIONS.

SUN'S ZENITH DISTANCE.			BAROMETER.		
Z. D.	No.	AC .	Reading.	No.	AC .
°		"	in.		"
81	1	1405.0	29.40	1	1403.0
82	..		29.50	2	1380.5
83	..		29.60	10	1384.6
84	2	1398.0	29.70	22	1383.9
85	1	1398.0	29.80	30	1389.8
86	..		29.90	20	1401.9
87	3	1397.3	30.00	16	1409.7
88	4	1398.0	30.10	6	1396.7
89	5	1396.2	30.20	3	1398.7
90	3	1403.7	30.30	2	1405.5
91	5	1402.6	30.40	5	1405.6
92	4	1409.2	30.50	..	
93	4	1411.5	30.60	8	1403.2
94	4	1396.0	30.70	5	1409.0
95	2	1394.5			
96	1	1450.0			
THERMOMETER.			BESSEL'S REFRACTIONS.		
Reading.	No.	AC .	Log.	No.	AC .
°		"			
-10	5	1410.4	-.030	35	1378.9
0	20	1403.5	-.020	30	1389.0
+10	5	1407.2	-.010	15	1386.3
+20	10	1411.9	.000	3	1377.3
+30	6	1408.2	+.010	6	1395.5
+40	4	1401.5	+.020	3	1407.3
+50	5	1380.2	+.030	13	1416.5
+60	8	1390.0	+.040	3	1399.7
+70	20	1384.8	+.050	15	1400.9
+80	28	1389.4	+.060	7	1413.7
+90	24	1374.5	+.070	5	1410.4

The first part of this table shows that no appreciable effect is produced on the refraction by the varying altitude of the sun. This conclusion was also derived from the observations on Wachusett. The group corresponding to a zenith distance of 96° consists of a single set, No. 12. The separate settings are discordant, the observation was very uncertain on account of the darkness, and *B* had become invisible. A variation in the refraction is obvious when either the barometer or thermometer varies. Low temperatures are, however, accompanied by high pressures, as is shown in the mean values given on page 282. The effect of these two causes cannot readily be distinguished. Probably both act, since both affect the density of the air. The grouping according to both temperature and pressure shows that this correction is a real one, and should be applied, although this is not customary in geodetic work.

TABLE VII.—CONSTANTS OF INSTRUMENT.

Date.	G. M. T.	Level.	Coll.	Alt.	Res.	Obj.
1882.	h.	"	"	"	"	
August 6	2.7	+19	+128	— 320	—3	<i>C</i>
"	3.4	+12	+126	— 322	—5	<i>C</i>
"	7.3	+29	+125	— 313	+4	<i>C</i>
"	11.4	+12	+133	+1083	...	<i>A</i>
"	"	+14	+129	+ 688	...	<i>B</i>
"	"	+14	+128	— 318	—1	<i>C'</i>
"	21.3	+ 1	+136	+ 681	...	<i>B</i>
"	"	0	+136	— 314	+3	<i>C'</i>
"	22.2	+ 2	+131	+ 687	...	<i>B</i>
"	"	+ 4	+131	— 311	+6	<i>C'</i>
"	23.2	+ 7	+132	+1078	...	<i>A</i>
"	"	+ 9	+132	+ 680	...	<i>B</i>
"	"	+ 7	+132	— 317	0	<i>C</i>
"	"	+ 8	+132	— 315	+2	<i>C'</i>
"	23.7	+11	+129	+1076	...	<i>A</i>
"	"	+ 6	+133	+ 680	...	<i>B</i>
"	"	+ 5	+133	— 315	+2	<i>C</i>
August 7	9.4	+ 9	+133	+1063	...	<i>A</i>
"	"	+ 7	+135	+ 680	...	<i>B</i>
"	"	+ 7	+135	— 323	—6	<i>C'</i>
"	10.1	+ 8	+129	+1068	...	<i>A</i>
"	"	+ 5	+130	+ 682	...	<i>B</i>
"	"	+ 3	+127	— 322	—5	<i>C'</i>

A series of measurements was made on August 6 and 7 to determine the steadiness of the constants of the instrument. In Table VII. the successive columns give the date, the Greenwich mean time, the level, the collimation, and the absolute altitude of the point observed,

in seconds of arc. The mean altitude of the nearer point, C , was found by these observations to be $-317''$, and the next column gives the residual from this mean. The last column gives the object observed, C' being employed to indicate a second point on C , which with certain illuminations was more readily seen. C' was found to be $277''$ below C , and the measures of its altitude have been corrected by that amount.

The average value of the residuals in the last column but one is $3.''4$, which shows the degree of accuracy that may be attained with this instrument in determining absolute altitudes. The accuracy, portability, and cheapness of the micrometer level ought to render it useful for many purposes. Valuable work could be done with it from any station commanding a distant view. Observations of distant points in different azimuths, under varying meteorological conditions, are still much to be desired.

From the differential character of the observations described above, it has not been necessary to consider the curvature of the earth, or its variation in different azimuths, in the present discussion.